Stick slip vibrations in drilling:
Modeling, avoidance and open problems

Ulf Jakob F. Aarsnes
ulaa@norceresearch.no

Norwegian Research Center (NORCE)

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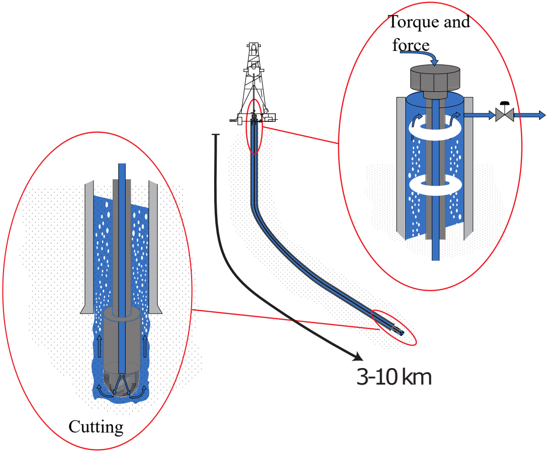
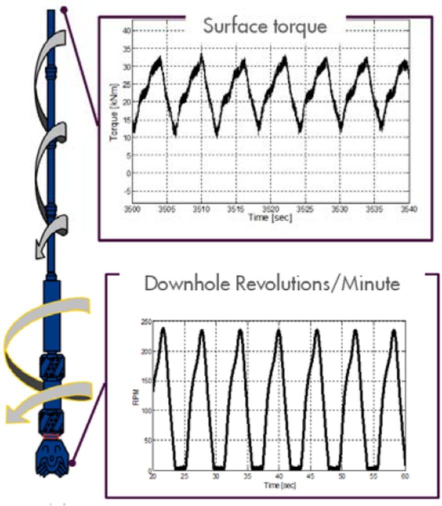
1. Stick slip phenomenology and causes
2. Mathematical model
   - Distributed drill string model
   - Bit-rock regenerative effect
   - Off-bottom vibrations and side forces
3. Avoidance: Industrial Controllers
4. Current state and open problems

Drilling

- Wells are drilled up to 10 km long
- Force and torque transfered from top to bit



Stick slip



- Unwanted torisional oscillations (cyclic sticking and slipping)
- 3-10 second period (dependent on drillstring length)
- Reduces effectiveness, causes damage
- Seen topside as torque fluctuations

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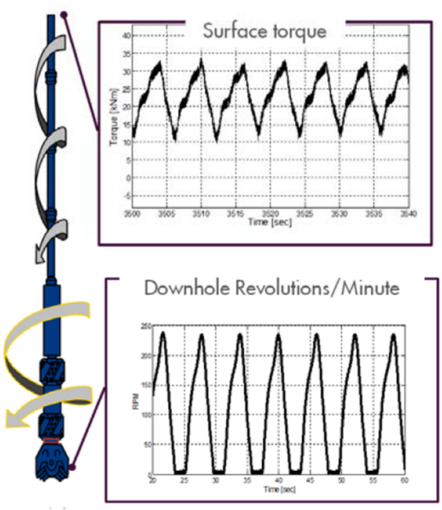

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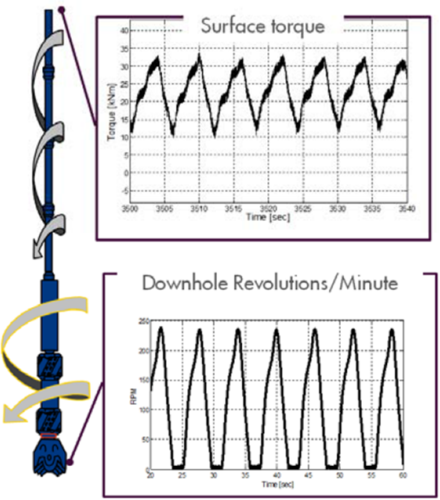

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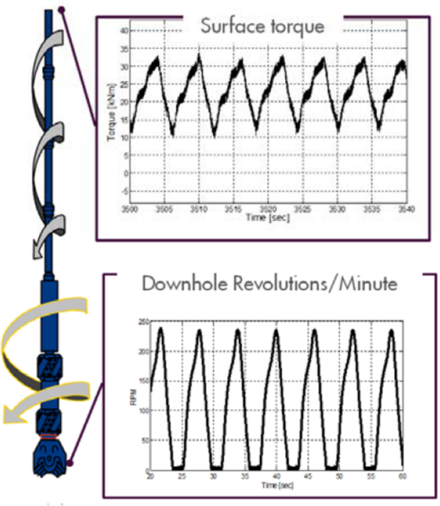

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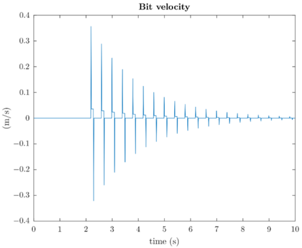
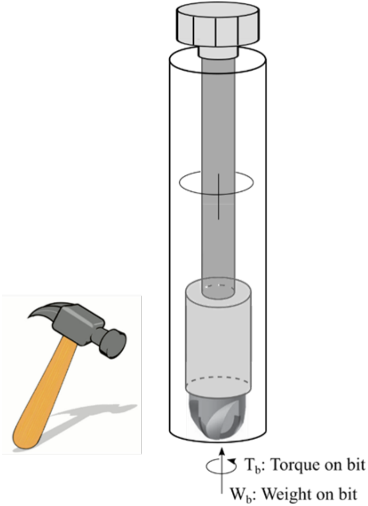
NOT free Oscillations

- System response to impulse shock/disturbance
- No new energy enters system. Oscillations die out over time





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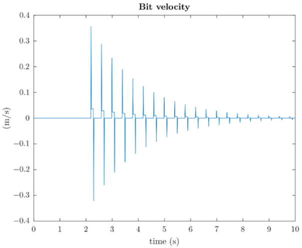
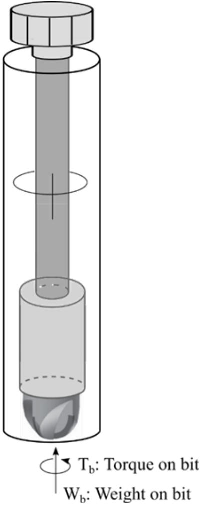
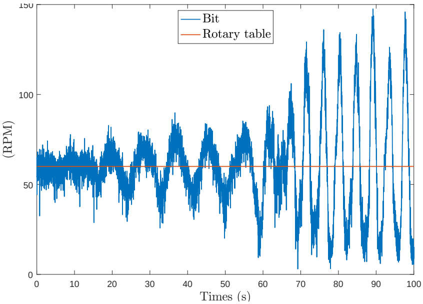




Self excited vibrations

Sustained stick slip *must* be caused by an unstable equilibrium in the process dynamics:

1. Regenerative effect in bit-rock interaction (left).
2. Velocity weakening effect in side forces (right).



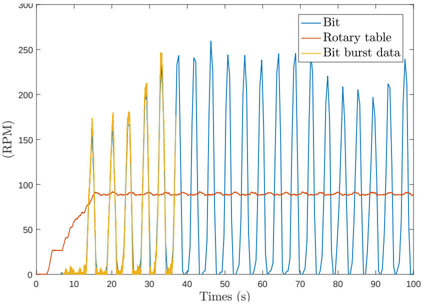
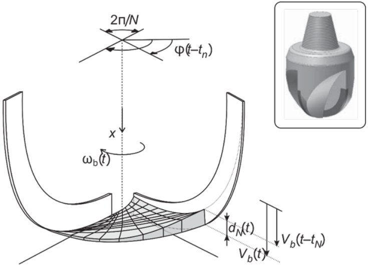


Figure: Field examples of stick-slip.

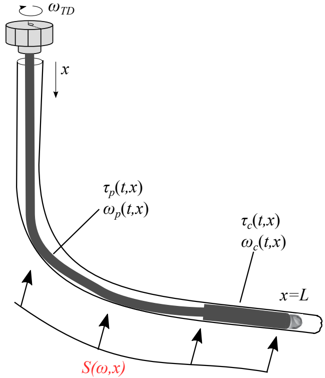
The regenerative effect:

- Well known from machine tooling (cutting) processes
- Proposed by [Detournay, E and Defourny, P 1992] to be cause of stick slip in drilling
- Effect experimentally verified in cutting processes



Velocity weakening of side forces

- Stick slip off-bottom: no bit rock interaction. Need different explanation
- **Side force:** Interaction between drill string and borehole
- **Velocity weakening:** Reduced force with increased velocity



Goal of presentation

1. Model these two causes of stick slip
2. Discuss mitigation techniques
3. Point out further required improvements

Case for distributed model

Drilling vibrations have a wide frequency spectrum.

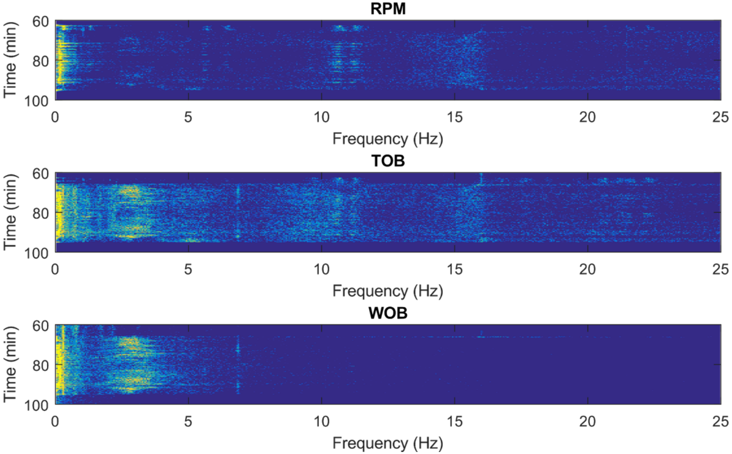


Figure: Spectrogram of field data

Case for distributed model

Lumped models have limited applicability.

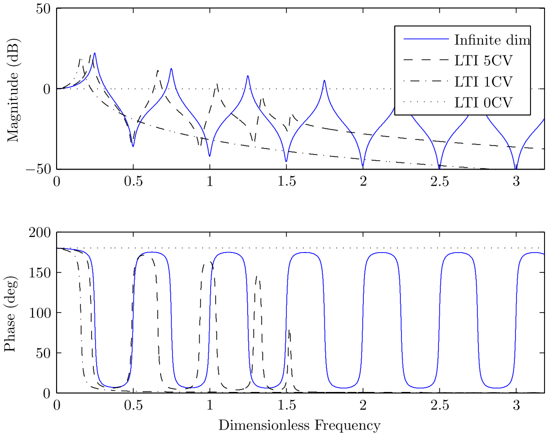
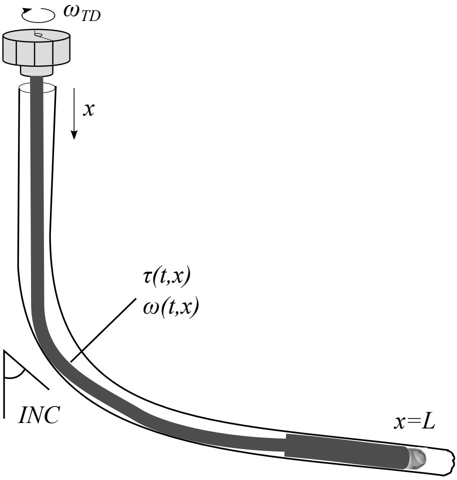


Figure: Frequency domain comparison of lumped vs distributed model.

Distributed Model: Torsional Drill string dynamics



Topside BC

$$\dot{\omega}_{TD} = \frac{1}{I_{TD}}(\tau_m - \tau(0,t))$$

Distributed wave eq.: $i \in \{p, c\}$

$$\frac{\partial \tau_i(t,x)}{\partial t} + J_i G \frac{\partial \omega_i(t,x)}{\partial x} = 0$$

$$J_i \rho \frac{\partial \omega_i(t,x)}{\partial t} + \frac{\partial \tau_i(t,x)}{\partial x} = 0,$$

Coupling

$$\omega_p(L_p, t) = \omega_c(0, t)$$

$$\tau_p(L_p, t) = \tau_c(0, t)$$

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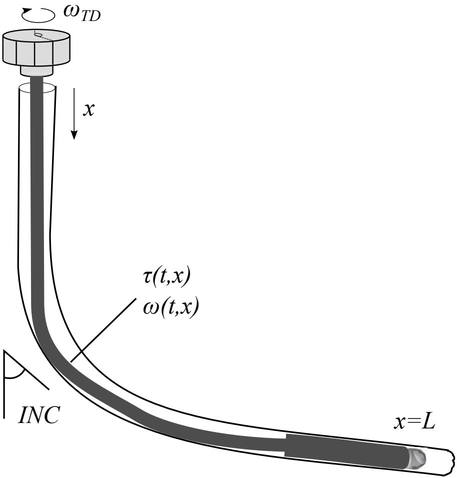

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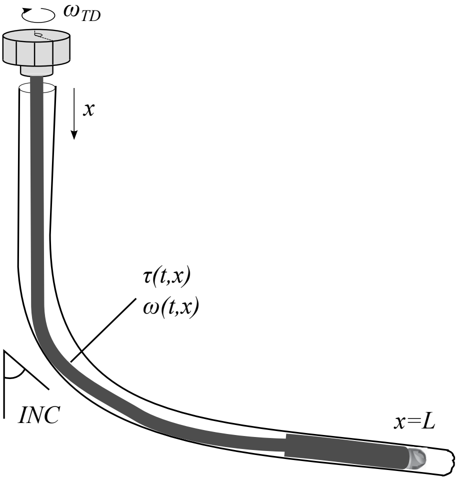

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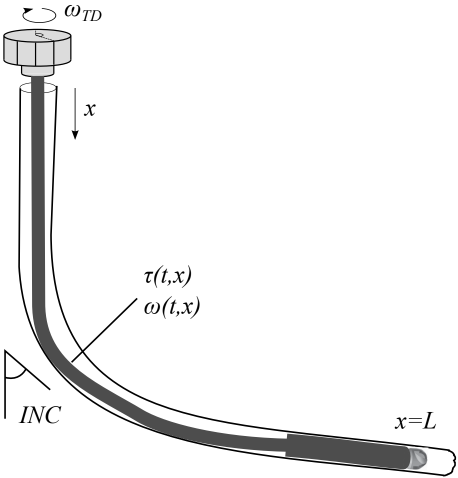

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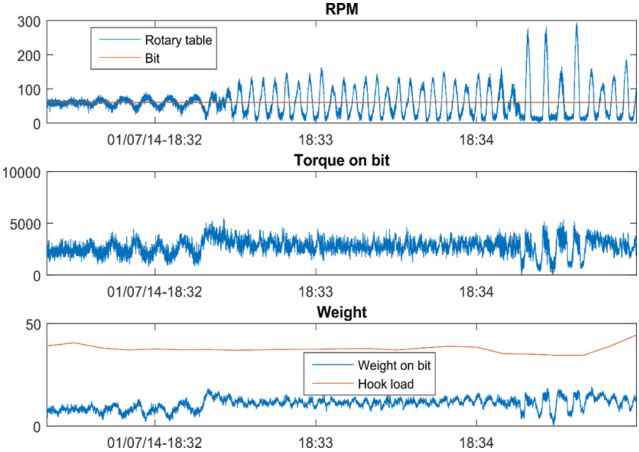
Field data example

- Example of stick slip caused by increased WOB.
- Increase in WOB makes equilibrium unstable.
- Not explained by static Coulomb friction!



Bit rock interaction [Detournay, E and Defourny, P 1992, Richard et al., 2007]

Relate bit position to weight and torque on bit.

- Depth of cut:

$$d(t) = N[X_b(t) - X_b(t - t_N(t))]$$

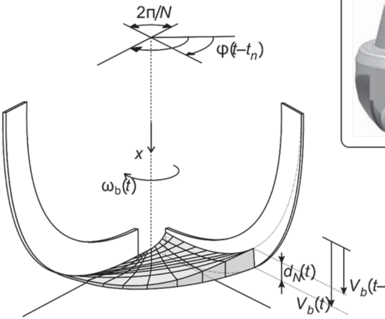
- Delay between two cutters:

$$\phi_b(t) - \phi_b(t - t_N(t)) = \frac{2\pi}{N}$$

- Torque and weight on bit:

$$W_b(t) = a\zeta\epsilon d(t) + W^*$$

$$W_b(t) = \frac{1}{2}a^2\epsilon d(t) + T^*$$



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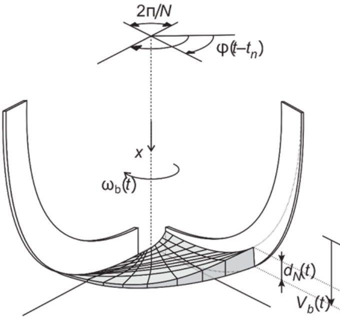

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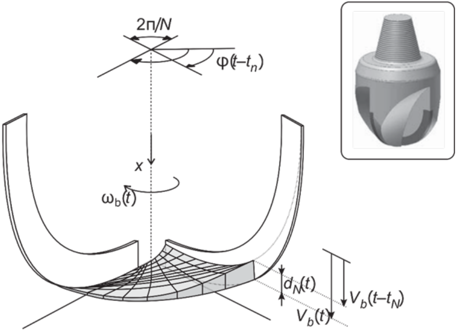

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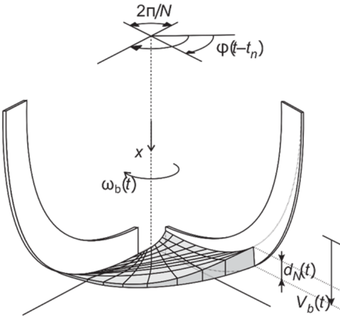

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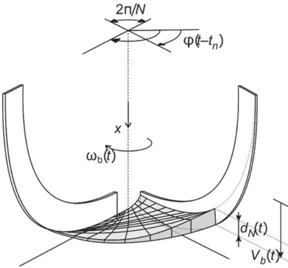
$$d(t) = N[X_b(t) - X_b(t - t_N(t))]$$

- Linearization:

$$d(t) \approx N[X_b(t) - X_b(t - t_N)] - \frac{v_0}{\omega_0}(\phi_b(t) - \phi_b(t - t_N))$$

- Solution in the frequency domain:

$$D(s) = \frac{N}{s}\left[V_b(S)(1 - e^{-st_N}) - \frac{v_0}{\omega_0}\Omega_b(s)(1 - e^{-st_N})\right]$$



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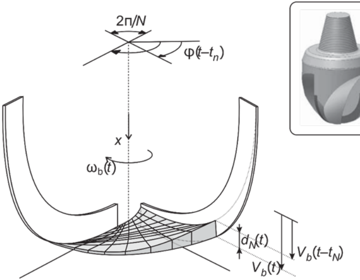

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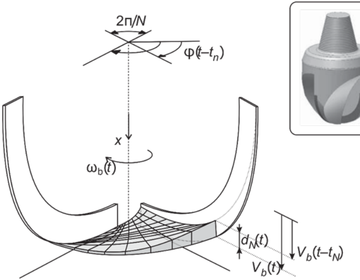
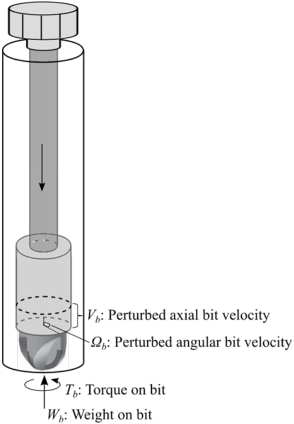


Drill string transfer function

- Employ transfer function description of drill string

$$\frac{V_b}{W_b} = -\frac{1}{\zeta_a} g_a(s)$$
$$\frac{\Omega_b}{T_b} - -\frac{1}{\zeta_t} g_t(s)$$

ζ_a, ζ_t are axial and torsional characteristic line impedances.

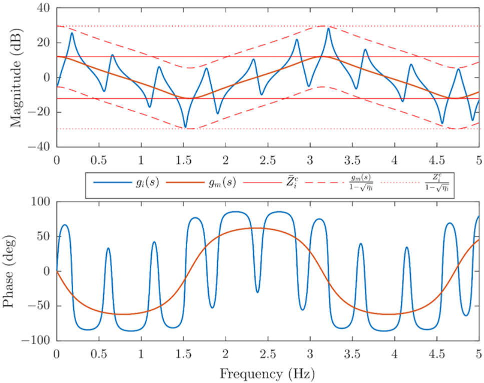


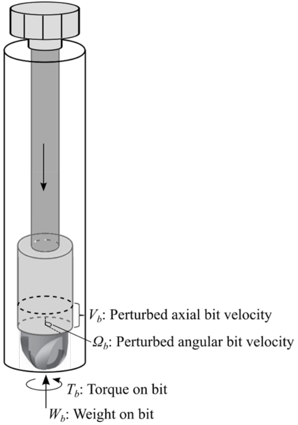
Drill string transfer function: Two sections

Drill string transfer function $g_i(s)$, $i \in t, a$ is determined by:

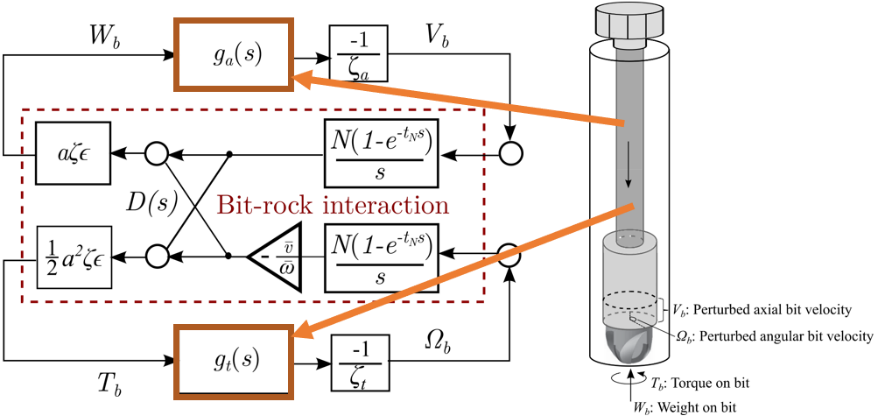
- Travel time: $t_i = L/c_i$
- Reflection coefficient: $R_i = \frac{Z_L - \zeta_i}{Z_L + \zeta_i}$

For pipe and collar section.





Self excited vibration feedback [Aarsnes, UJF., van de Wouw, N. 2018]



Characteristic equation

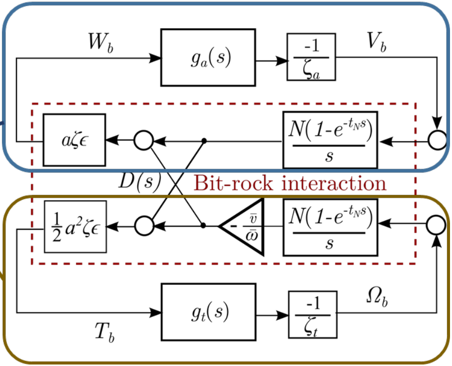
The characteristic equation consists of two *weakly* coupled loops:

$$G(s) = G_a(s) + G_t(s)$$

These can be used to determine *linear* stability.

$$G_a(s) = -g_a(s)\frac{K_a}{s}(1 - e^{-st_N})$$

$$G_t(s) = g_t(s)\frac{K_t}{s}(1 - e^{-st_N}).$$



Simulations [Aarsnes, UJF., van de Wouw, N. 2019]

Linear stability analysis reveals, for typical parameters:

1. Axial loop *is unstable*
2. Torsional loop sometimes unstable

Typical simulation examples without (left), and with stick slip (right):

What is the effect of the coupling?

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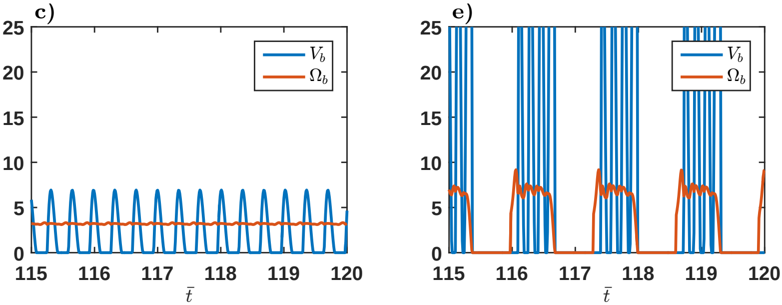
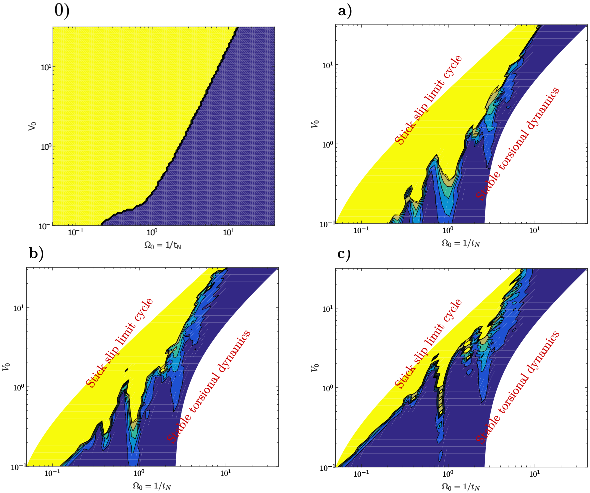


What is the effect of the coupling?

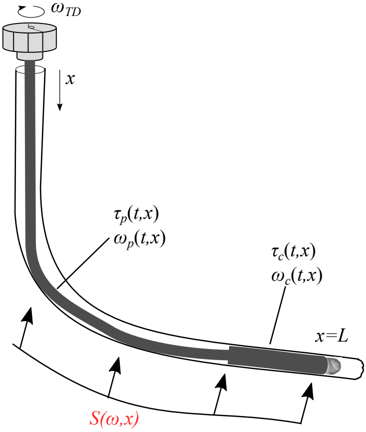
Stability map from simulations [Aarsnes, UJF., van de Wouw, N. 2019]

Axial *instability* increases torsional *stability*.



0) Linear ($K_a = 0$), **a)** $K_a = 10$, **b)** $K_a = 20$, **c)** $K_a = 40$.

Side force

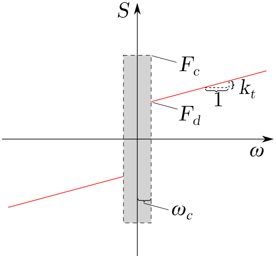


Assume no bit-rock interaction:
Rotation off bottom.

- Distributed wave eq.: $i \in \{p, c\}$

$$\frac{\partial \tau_i(t,x)}{\partial t} + J_i G \frac{\partial \omega_i(t,x)}{\partial x} = 0$$
$$J_i \rho \frac{\partial \omega_i(t,x)}{\partial t} + \frac{\partial \tau_i(t,x)}{\partial x} = -S(\omega_i, x),$$

- Coulomb friction side force



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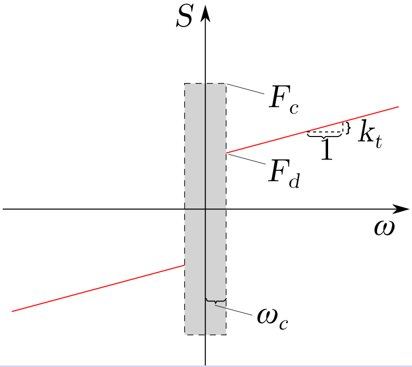
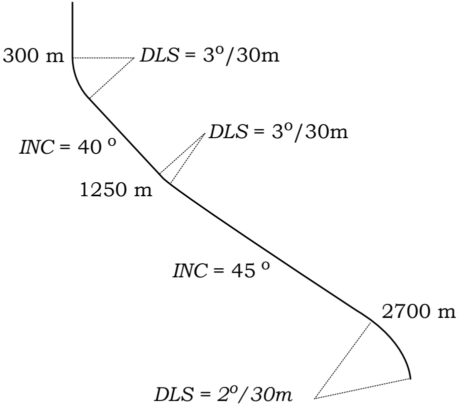


Coulomb friction as an inclusion:

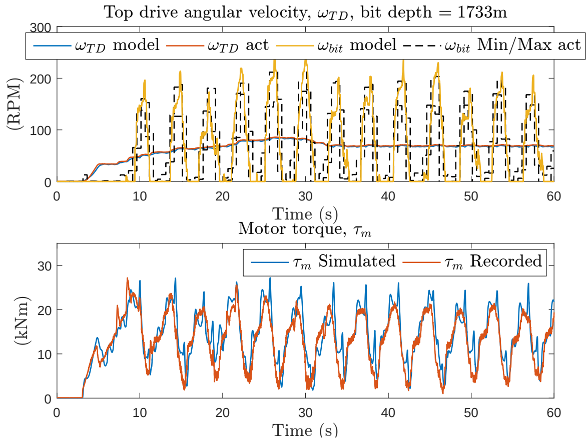
$$\begin{cases} S(\omega, x) = F_d(x), & \omega > \omega_c \\ S(\omega, x) \in [-F_c(x), F_c(x)] & |\omega| < \omega_c \\ S(\omega, x) = -F_d(x), & \omega < -\omega_c \end{cases}$$

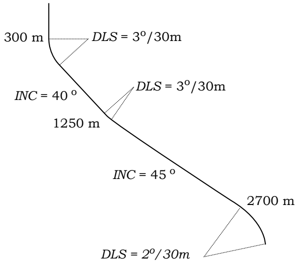
Simulation example

Simulation without bit-rock interaction.



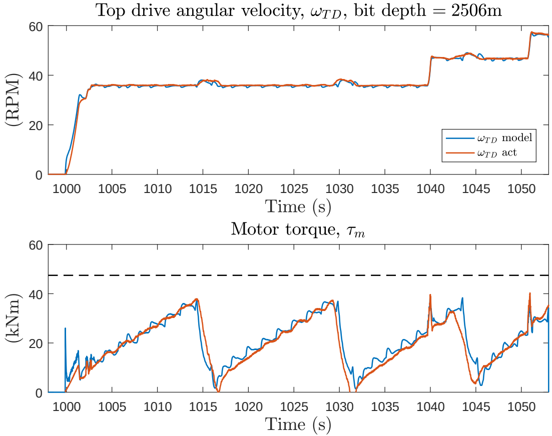
Field data ex 1. 1,733 meter [Aarsnes, UJF and Shor, RJ 2018]

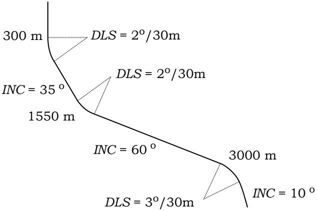




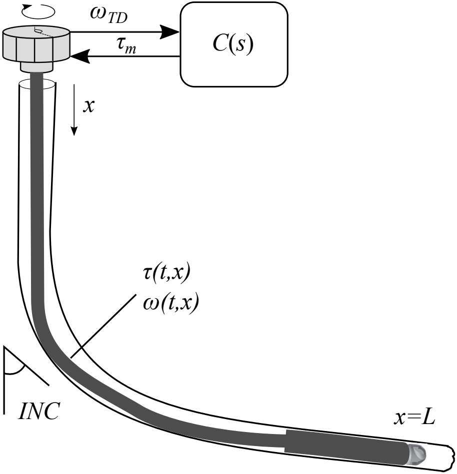
Comparison with off-bottom rotation (no bit-rock interaction).

Field data ex 2: 2,2506 meter [Aarsnes, UJF and Shor, RJ 2018]





Top drive control

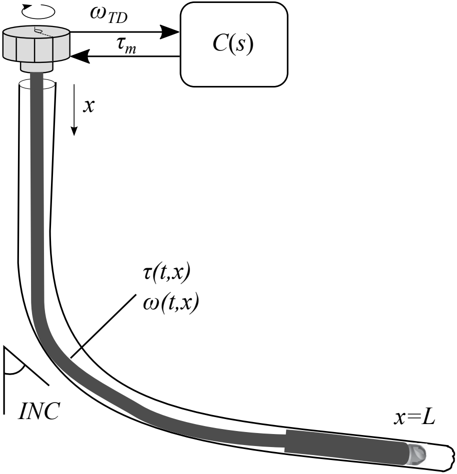


Topside BC

$$\dot{\omega}_{TD} = \frac{1}{I_{TD}}(\tau_m - \tau(0,t))$$

- Top drive is controlled by motor torque τ_m based on RPM measurements ω_{TD}.
- Control approach: Reduce the wave reflection.

Top drive control



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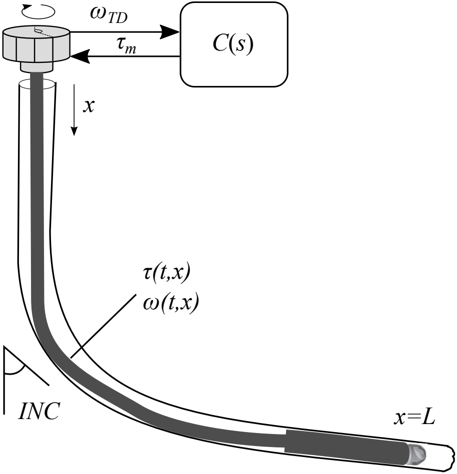

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Top drive control

- Topside BC (Laplace transformed):

$$s\omega_{TD} = \frac{1}{I_{TD}}(\tau_m - \tau(0))$$

- Top drive control

$$\tau_M = C(s)\omega_{TD}$$

- Then top drive impedance is:

$$Z_L(s) = \frac{\tau(0)}{\omega_{TD}}(s) = C(s) + I_{TD}s$$

- Wave reflection:

$$R(\omega) = \left| \frac{Z_L(j\omega) - \zeta_p}{Z_L(j\omega) + \zeta_p} \right|,$$

where ζ_p is pipe impedance.

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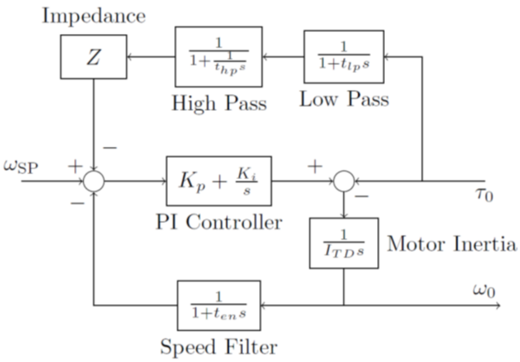
Stick slip mitigation by control [Kyllingstad, A. 2017]

Industrial available top drive speed control:

1. Stiff speed control
2. Tuned PI Control: SoftTorque/SoftSpeed
3. Impedance Matching: Ztorque



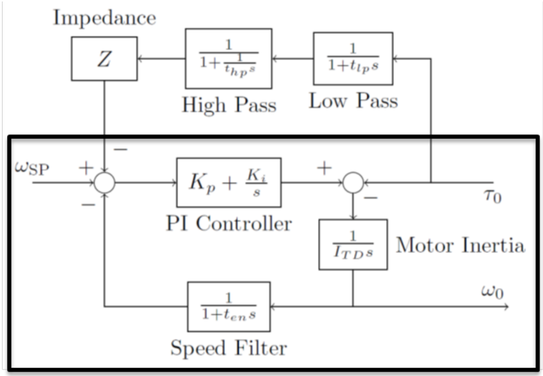
Stick slip mitigation by control

Top drive speed control:

1. **Stiff speed control**
   - $K_p = 100\zeta_p$
   - $K_i = 5I_{TD}$

   ζ_p is Pipe impedance.
   I_{TD} is top drive inertia.

- Top drive RPM tracks set-point.



Bit rock interaction

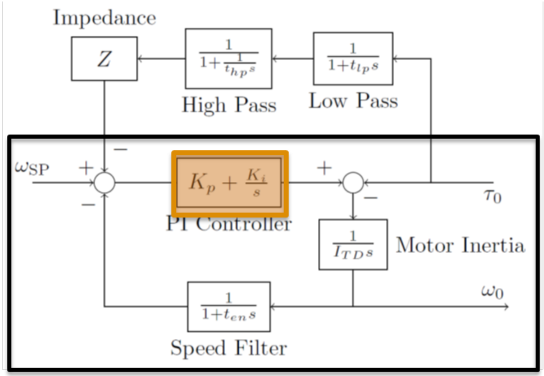
Top drive speed control:

2. **Tuned PI Control:** SoftTorque/SoftSpeed

   - $K_p = 4\zeta_p$
   - $K_i = (2\pi f_c)^2 I_{TD}^2$

   f_c is frequency of minimal reflectivity.

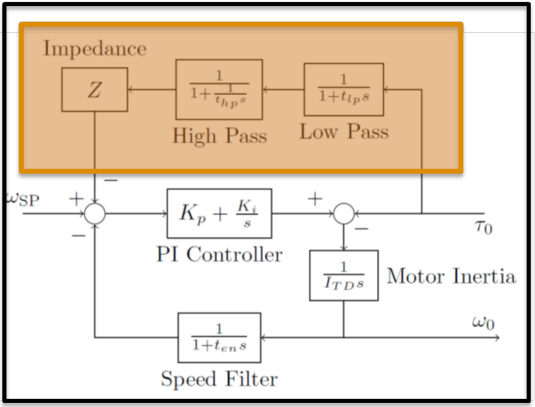
- Reduces reflection in limited frequency range.



Bit rock interaction

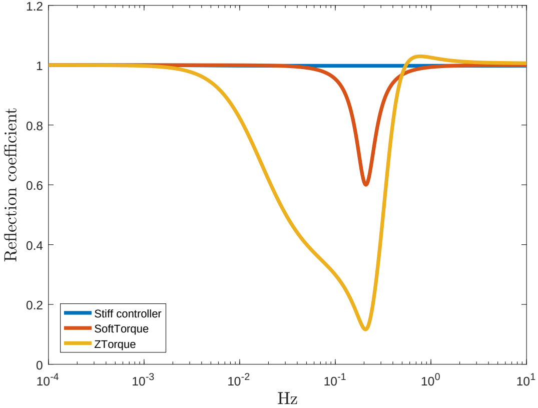
Top drive speed control:

3. **Impedance Matching:** Ztorque
   - At high frequencies: Top drive controlled to cancel reflections.
   - At low frequencies: Follow setpoint



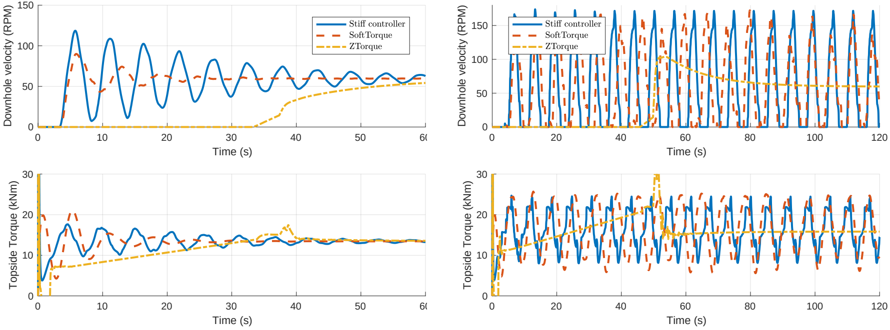
Reflectivity Comparison [Aarsnes, UJF. et al. 2018]

1. **Stiff Speed:** Full reflection
2. **SoftTorque/speed:** Limited reflection reduction
3. **ZTorque:** Improved reflection reduction. Limited by:
   - Tracking performance (filter cut-off)
   - Instrumentation (ideal case considered)

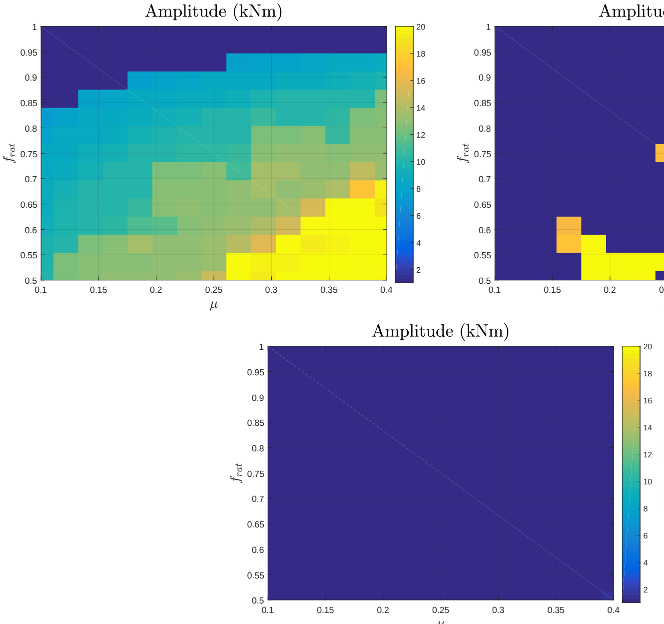


Stability map comparison: Off bottom model [Aarsnes, UJF. et al. 2018]

- Soft Torque/Speed works in some cases.
- Ztorque effective at avoiding stick slip, but yields slower control.

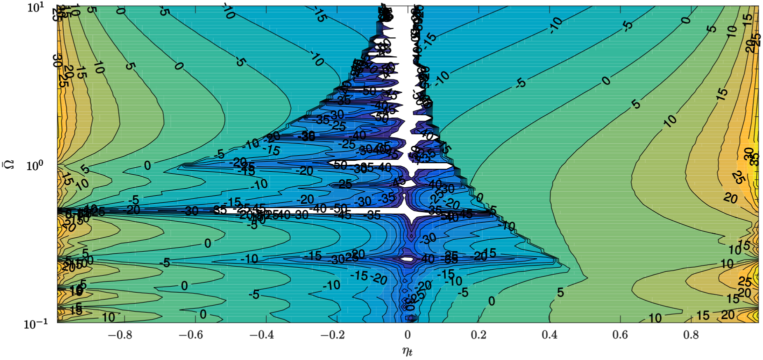


Simulation comparison [Aarsnes, UJF. et al. 2018]



Linear stability analysis: bit-rock interaction

- Higher numbers denote higer tendency to instability.
- X-axis denotes reflection coefficient.



Current state

- The cause and potential mitigation of stick slip is now quite well understood:
  1. Regenerative effect in the bit rock interaction
  2. Velocity weakening of the side forces
- Models capable of reproducing the phenomena.
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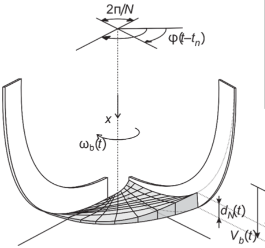
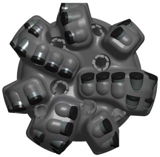
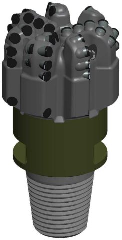
Open problems I

Modeling gap for the bit rock interaction to be useable in practice:



- Mathematical representation of a realistic PDC bit.
- Model stability maps should be tested and calibrated against experimental results and field data.
- **Goal:** To predict occurence, optimize operational parameters, improve bit design.





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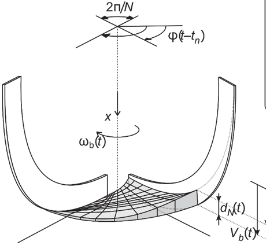

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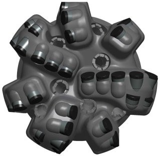

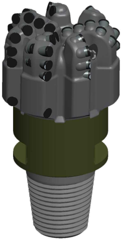

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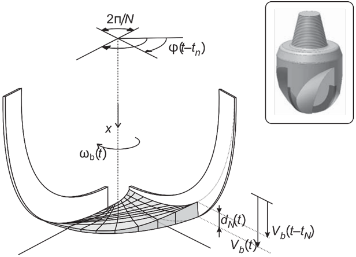

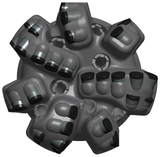

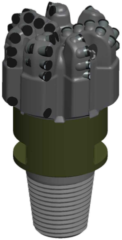

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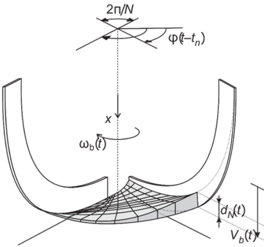
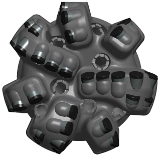
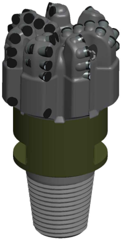
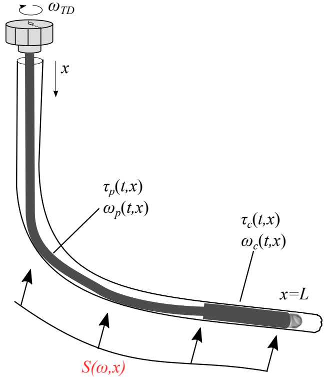






Open problems II

- Low predicitve power: Unkown friction factors for side forces.
- Comprehensive model need both side forces and bit-rock interaction.
- NOT COVERED: Lateral vibrations.



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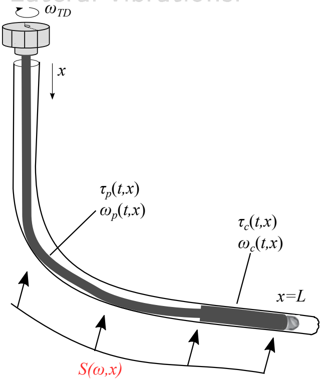

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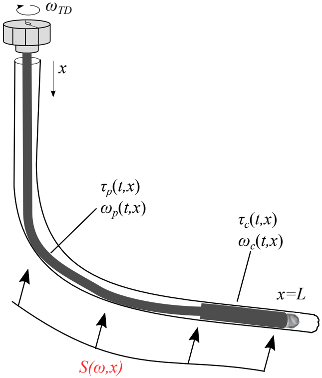

Summary I: Causeses and Modeling

- Two distinct known causes of torsional stick slip:
  1. Self regenerativ effect of the bit-rock interaction
  2. Velocity weakening in along string side forces
- Distributed (high order) models needed to have practical relevance.

Summary II: Current status and remaining challenges

- Remaining *key* modeling challenges:
  1. Make bit-rock interaction useable in practice
  2. Test model predictions against experimental and field data
  3. Model both side forces and bit-rock interaction.
  4. Understand coupling to lateral vibrations (whirl)
- Existing industrial controllers
  1. Approach: reduce reflection coefficient
  2. Effective, but limited by physical and instrumentation constraints
  3. Harder for larger top drives (high inertia).

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